Renormalization in the quasi parton distribution functions[†]

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In the study of high energy scattering processes, such as deep inelastic scattering (DIS) and the Drell-Yan process, one of the key concepts is "QCD collinear factorization," used to separate the perturbative and nonperturbative parts. The scattering cross sections are written as a convolution of the perturbative hard part and nonperturbative parton distribution functions (PDFs), which absorb all collinear divergences of the partonic scattering. PDFs are universal functions and thus are used to predict the cross sections of various hadronic scattering processes.

Direct calculation of the PDF by lattice QCD could give us information complementary to global QCD analyses. However, calculation of the light-cone distributions on the Euclidean lattice is difficult because of its time-dependence. Ji recently proposed the quasi-PDF approach to solve this problem.[1)] The quasi-PDFs are defined with fields correlated completely along the spatial direction; for example,

$$\widetilde{q}(\tilde{x},\mu,P_z) = \int \frac{d\delta z}{4\pi} e^{-i\delta z\tilde{x}P_z} \times\langle P_z|\overline{\psi}(\delta z)\gamma^3 \exp\left(-ig\int_0^{\delta z} dz' A_3(z')\right)\psi(0)|P_z\rangle \quad (1)$$

for the quasi-quark distribution, where the hadron state has a large momentum in z-direction P_z. Because of the time-independence, the quantity in Eq. (1) is calculable on the Euclidean lattice. The quasi-distributions are matched with usual light-cone distributions through the large momentum effective theory (LaMET).[2)] After the quasi-distribution approach was introduced, several lattice QCD calculations of quasi-quark distributions have been carried out. These studies are still exploratory, and thus there are many uncertainties. Among the uncertainties, we address power divergences and the matching between the continuum and lattice in this article.

Unlike the original light-cone PDFs, quasi-distributions are known to have power divergences that originate from a Wilson line in the operator definition. The power divergences must be taken into account in the matching procedure. The renormalization of the non-local quark bilinears of the hadronic matrix element in the r.h.s. of Eq. (1), called $O(\delta z)$, is assumed to be[3)]

$$O(\delta z) = Ze^{\delta m|\delta z|}O^{\rm ren}(\delta z), \quad (2)$$

where a superscript "ren" indicates that the operator is renormalized, Z contains logarithmic divergences, and δm denotes the mass renormalization of a test particle moving along the straight line connecting the two

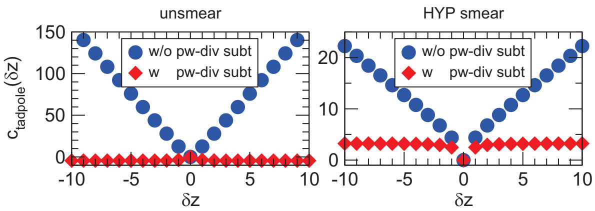

Fig. 1. Tadpole diagram contribution of the matching factor to the one-loop coefficient.

quark fields. Moreover, δm in the exponential factor takes care of all the power divergence, which is valid nonperturbatively. However, the multiplicative renormalizability of the logarithmic divergence is proven only up to the two-loop level.[4)] While the all-order proof of the multiplicative renormalizability is important to give a solid validation to the renormalization in the quasi-distribution approach,[5)] we assume the form in Eq. (2) for now. Knowing the renormalization in Eq. (2), the power divergence subtraction can be performed by defining a modified non-local operator for Eq. (1):[6)]

$$O^{\rm subt}(\delta z) = e^{-\delta m|\delta z|}O(\delta z), \quad (3)$$

where the superscript "subt" indicates that this is a power divergence subtracted operator. The mass renormalization δm now needs to be fixed by imposing a renormalization condition. For this fixing, we can use static potential $V(R)$ and set $V(R_0) = V_0$ as the condition, leading to $2\delta m = V_0 - V(R_0)$.

By removing the power divergence, we can perform continuum-lattice matching. We demonstrate the one-loop matching using the lattice perturbation theory with a naive fermion and plaquette gluon. The one-loop matching factor is expressed as

$$Z_{\rm cont-latt}(\delta z) = 1 + \frac{g^2}{(4\pi)^2}C_F c(\delta z). \quad (4)$$

In the continuum, we introduce a UV cutoff scale and set to the lattice cutoff $\mu = a^{-1}$. As an example, we show the tadpole diagram contribution in Fig. 1 with and without Wilson line smearing. The smearing helps to reduce the difference between the continuum and lattice caused by the power divergence before the subtraction. Further study on the matching using the nonperturbative method is underway.

References

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