

Canonical approach to finite density QCD with multiple precision computation[†]

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Although QCD has a rich phase structure at a finite temperature and density,¹⁾ investigations based on first-principles calculations are limited in small density regions. This is because the action with a finite real quark chemical potential μ_q is complex in general and the Monte Carlo method with the complex action does not work. This problem is called the sign problem.²⁾ In this work, the canonical approach³⁾ as a method for finite density QCD is studied. In the canonical approach, the grand canonical partition function $Z_{GC}(\mu_q)$ at a finite real quark chemical potential is represented as the fugacity expansion with the canonical partition function Z_n as follows:

$$Z_{GC}(\mu_q) = \sum_{n=-\infty}^{\infty} Z_n e^{n\mu_q/T}. \quad (1)$$

The canonical partition function is given by the Fourier transformation of the grand canonical partition function calculated at a purely imaginary chemical potential as follows:

$$Z_n = \frac{1}{2\pi} \int_0^{2\pi} d\left(\frac{\mu_I}{T}\right) Z_{GC}(i\mu_I) e^{-in\mu_I/T}, \quad (2)$$

where μ_I is a real number. This procedure to obtain the canonical partition functions does not suffer from the sign problem because the action with purely imaginary chemical potential is real and the Monte Carlo method works. This is a strong advantage of the canonical approach. However, we encounter some difficulties as follows. In the calculation to obtain the canonical partition function at a large baryon number, the Fourier transformation becomes a highly oscillating integral. Thus, it is difficult to numerically compute the integral because of cancellation of significant digits in the Fourier transformation. To avoid this problem, the multiple precision computation⁴⁾ is adopted in this work. Another problem is the numerical cost for the calculation of the grand canonical partition function at a purely imaginary chemical potential. To perform the Fourier transformation of the grand canonical partition function, we need the fermion determinants at many different values of the purely imaginary chemical potential. To reduce this numerical cost, the winding number expansion based on the hopping parameter

expansion for the Wilson fermion formalism is developed in this work. In this method, we can expand the fermion determinant $\det D(\mu)$ with the complex coefficients $\{W_n\}$ as follows:

$$\log \det D(\mu) = \sum_{n=-\infty}^{\infty} W_n e^{n\mu/T}, \quad (3)$$

where μ is a complex number. Therefore, if we calculate the coefficients at some quark chemical potential once, we can obtain the fermion determinant at any quark chemical potential.

Figure 1 shows the baryon chemical potential μ_B dependence of the pressure evaluated by the canonical approach and the direct method. We directly use the winding number expansion of Eq. 3 for the calculation of the pressure at a real baryon chemical potential in the direct method. The upper bounds in this figure correspond to the validity range of these methods. Therefore, we can conclude that the validity range of the canonical approach is wider than that of the direct method.

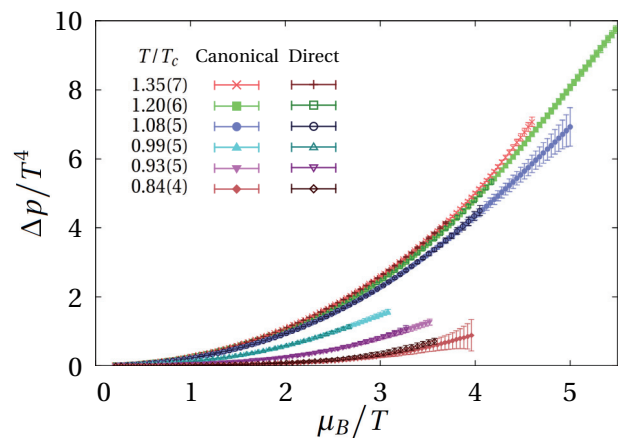


Fig. 1. The baryon chemical potential μ_B dependence of the pressure evaluated by the canonical approach and the direct method: Δp is defined as $\Delta p = p(\mu_B/T) - p(0)$.

References

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