Nucleon form factors with large momenta from lattice QCD

S. Syritsyn[*1,*2,*3] and B. Musch[*3,*4]

Vector form factors are among the most important features of a nucleon describing its distributions of charge and magnetization densities,

$$\langle P+q|\bar{q}\gamma^\mu q|P\rangle = \bar{u}_{P+q}\big[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}}{2M_N}\big]u_P\,,$$

where $F_{1,2}$ are Dirac and Pauli form factors, respectively, and $Q^2 = -q^2 > 0$ is the momentum transfer squared in elastic nucleon-photon scattering. Although the proton and neutron form factors have been studied extensively, many puzzles remain. The behavior of form factors at high momenta is expected to transition to perturbative QCD scaling,[1,2] which has not been observed yet. The ratio the proton electric and magnetic form factors may cross zero around $Q^2 = 7\ldots 8\,\mathrm{GeV}^2$, placing constraints on nucleon models in the Dyson-Schwinger framework. There is a large qualitative difference of contributions between u- and d-quarks to both the F_1 and F_2 form factors[3]; understanding this phenomenon may also shed light on the internal organization of the nucleons. An upcoming experimental program at JLab will measure the nucleon form factors up to $Q^2 \approx 18\,\mathrm{GeV}^2$.

The study of high-momentum nucleon states on a lattice is complicated by statistical noise from cancellations and reduced energy gaps between the ground and excited states. A novel technique to study boosted nucleon states on a lattice has been recently introduced,[4] in which the quark fields used to construct the nucleon field are spatially "smoothed" with the momentum-translated covariant smearing operator,

$$\mathcal{S} = e^{\frac{1}{4}w^2\Delta} \to e^{+i\vec{k}_0\vec{x}}e^{\frac{1}{4}w^2\Delta}e^{-i\vec{k}_0\vec{x}}, \qquad (1)$$

where Δ is the gauge-covariant lattice Laplacian. Such an operator leads to a substantial improvement of the signal-to-noise ratio for boosted nucleon correlators.[4] We select $\vec{k}_0 = \frac{1}{3}\vec{P}$ assuming that quarks carry equal fractions of the nucleon momentum, although a larger value may be optimal.[4]

We have performed initial calculations on a $32^3\times 64$ lattice with $N_f = 2+1$ dynamical flavors of Wilson-clover quarks, lattice spacing $a = 0.091(2)\,\mathrm{fm}$ and pion mass $m_\pi = 278(3)\,\mathrm{MeV}$. With approximately 15,000 Monte Carlo samples, we can calculate nucleon-current correlators with source-sink separation up to $t_{\mathrm{sep}} = 10a = 0.9\,\mathrm{fm}$. Such a t_{sep} may be insufficient to suppress excited states, and we perform 2-state fits to extract matrix elements between the nucleon ground states. In the figure we show the ratio of the proton Pauli and Dirac form factors Q^2F_2/F_1 with the additional factor Q^2 compensating for the diffference in asymptotic scaling.[2] Quantum corrections modify the scaling by a factor $\log(Q^2/\Lambda^2)$,[1] which agrees with a recent analysis of JLab data.[3] Our lattice results agree qualitatively with this behavior. Different curves correspond to different $t_{\mathrm{sep}} = \{8,9,10\}a$ and different methods to extract the ground state. Remarkably, although the form factors $F_{1,2}$ are likely affected by excited states, their ratio does not change when excited states are suppressed by increasing t_{sep}. This is expected because perturbative QCD should not be able to discriminate between the ground and excited states of the nucleon, which are essentially nonperturbative, and the form-factor ratio may be universal for the ground and excited states.

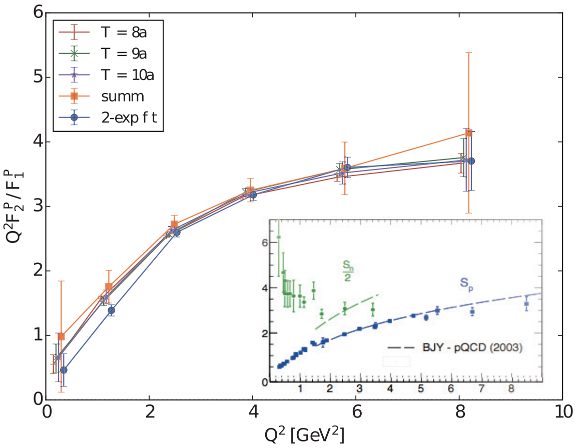

Fig. 1. Scaling of nucleon form factors with momentum transfer Q^2, lattice and phenomenology.[1]

Currently, this calculation is extended to include quark-disconnected contributions, variational analysis of excited states, improvement of the lattice quark vector current, selection of the optimal value for $\vec{k}_0$,[4] and study of the continuum limit. The calculation of high-momentum form factors from first principles will enable the comparison of the fundamental theory (QCD) and on-going experiments. In addition, it will validate the methodology for studying high-momentum nucleon states on a lattice. This method will be important for lattice calculations of TMDs and PDFs also requiring high-momentum nucleon initial and final states.

*1 RIKEN Nishina Center
*2 Department of Physics and Astronomy, Stony Brook University
*3 Institüt für Theoretische Physik, University of Regensburg
*4 Theory Group, Jefferson Lab

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