Low-lying collective excited states in non-integrable pairing models based on stationary phase approximation to the path integral†

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The time-dependent mean-field (TDMF) theory is a standard theory to describe the dynamics of nuclei from the microscopic degrees of freedom.[1)] Inclusion of the pairing dynamics leads to the time-dependent Hartree-Fock-Bogoliubov (TDHFB) theory, which has been utilized for a number of studies of nuclear reaction and structure. The small-amplitude approximation of TDHFB with modern energy density functionals, namely quasiparticle random phase approximation (QRPA), has successfully reproduced the properties of giant resonances in nuclei. In contrast, the QRPA description of low-lying quadrupole vibrations is not as good as that of the giant resonances. A large-amplitude nature of the quantum shape fluctuation is supposed to be important for these low-lying collective states.

The TDMF (TDHFB) theory corresponds to an SPA solution in the path integral formulation. It lacks a part of quantum fluctuation, which is important in large amplitude dynamics. To introduce the quantum fluctuation based on the TDHFB theory, requantization is necessary. A simple and straightforward way of requantization is canonical quantization. This is extensively utilized for collective models in nuclear physics.

In our previous work,[2)] we studied various requantization methods for the two-level pair model, in order to investigate the low-lying excited 0^+ states. Since the collectivity is rather low in the pairing motion in nuclei, the canonical quantization often fails to produce an approximate answer to the exact solution. In contrast, the stationary phase approximation (SPA) to the path integral can give quantitative results not only for the excitation energies but also for the wave functions and two-particle-transfer strengths. The quantized states obtained in the SPA has two advantages: First, the wave functions are given directly in terms of the microscopic degrees of freedom. Second, the restoration of broken symmetries are automatic. However, the applications of SPA have been limited to integrable systems. This is because we need to find separable periodic trajectories on a classical torus. Since nuclear systems, of course, correspond to non-integrable systems, a straightforward application of the SPA is not possible.

In this paper, we propose a new approach of SPA applicable to non-integrable systems, which is based on the extraction of the one-dimensional (1D) collective coordinate using the adiabatic self-consistent collective coordinate (ASCC) method.[1)] Since the 1D system is integrable, the collective subspace can be quantized with the SPA. The optimal degree of freedom associated with a slow collective motion is determined self-consistently inside the TDHFB space, without any assumption. Thus, our approach of ASCC+SPA basically consists of two steps: (1) Find a decoupled 1D collective coordinate of the collective motion of interest. (2) Apply the SPA to the collective mode.

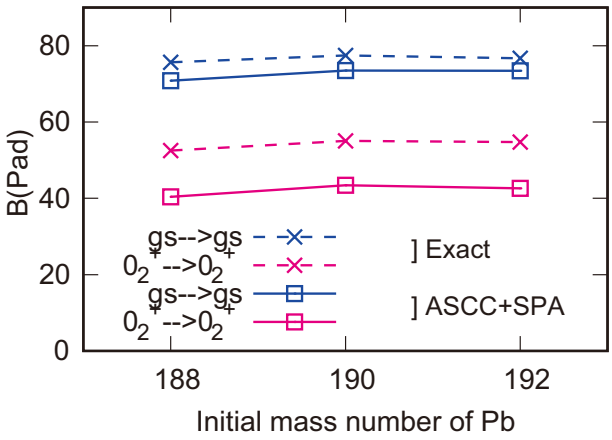

Fig. 1. Calculated two-neutron transfer matrix elements for Pb isotopes.

On the decoupled collective subspace, which is derived by the ASCC, we apply the Einstein-Brillouin-Keller (EBK) quantization rule,

$$\oint_{C_k} p_1 dq^1 = 2\pi k, \tag{1}$$

where k is an integer with a unit of $\hbar = 1$. Then, on this trajectory C_k, the k-th stationary state is given as a superposition of the (generalized) Slater determinants:

$$|\psi_k\rangle = \oint_{C_k} \rho(q,p) dt\, |q,p\rangle\, e^{i\mathcal{T}[q,p]}, \tag{2}$$

where $\mathcal{T}[q,p]$ is the classical action integral. This method is applied to the neutron pairing model for Pb isotopes. We calculate the two-neutron-transfer strengths from the ground to the ground and from the excited to the excited states (Fig. 1). A reasonable agreement is found between the ASCC+SPA and the exact calculations.

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