Double charge-exchange phonon states†

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The possibility of inducing double charge-exchange excitations by heavy-ion beams at intermediate energies has recently fostered interest in new collective excitations such as double isobaric analog states (DIAS) and double Gamow-Teller giant resonances (DGT). We present some formulas to evaluate different combinations of the average excitation energies of DIAS and DGTR by using commutator relations.

The expectation value for the energy of the DIAS (DGT) is defined as

$$E_{\rm DIAS(DGT)} \equiv \langle {\rm DIAS(DGT)}|\mathcal{H}|{\rm DIAS(DGT)}\rangle - \langle 0|\mathcal{H}|0\rangle. \quad (1)$$

Here $|0\rangle$ represents the ground state, and

$$|{\rm DIAS(DGT)}\rangle \equiv \frac{O_-|{\rm IAS(GT)}\rangle}{\langle {\rm IAS(GT)}|O_+O_-|{\rm IAS(GT)}\rangle^{1/2}} \quad (2)$$

is the definition of the DIAS (DGT) state in terms of the IAS (GT), which can be written as

$$|{\rm IAS(GT)}\rangle \equiv \frac{O_-|0\rangle}{\langle 0|O_+O_-|0\rangle^{1/2}}\,. \quad (3)$$

The operators $O_\pm = T_\pm \equiv \sum_i^A t_\pm(i)$ are the isospin raising and lowering operators for IAS and DIAS. For GT and DGT, $O_\pm = G_\pm \equiv \sum_i^A \sigma_z(i)t_\pm(i)$. Starting from Eq. (1), one may write the excitation energy of the DIAS (DGT) as

$$E_{\rm DIAS(DGT)} = \frac{\langle 0|[O_+^2,[\mathcal{H},O_-^2]]|0\rangle}{\langle 0|O_+^2O_-^2|0\rangle}, \quad (4)$$

assuming that the ground state has good isospin, that is, there is no isospin mixing and $T_+|0\rangle = 0$. Within the same approximation, $E_{\rm IAS(GT)}$ reads

$$E_{\rm IAS(GT)} = \langle {\rm IAS(GT)}|\mathcal{H}|{\rm IAS(GT)}\rangle - \langle 0|\mathcal{H}|0\rangle = \frac{\langle 0|[O_+,[\mathcal{H},O_-]]|0\rangle}{\langle 0|O_+O_-|0\rangle}. \quad (5)$$

One can write the energies for DIAS and DGT as

$$E_{\rm DIAS(DGT)} = 2E_{\rm IAS(GT)} + \frac{\langle 0|[O_+,[O_+,[[\mathcal{H},O_-],O_-]]]|0\rangle}{2(N-Z)(N-Z-1)}. \quad (6)$$

The denominator of Eq. (4) is evaluated to be $\langle 0|O_+^2O_-^2|0\rangle = 2(N-Z)(N-Z-1)$. The second term on the right-hand side in Eq. (6) could be different from zero only for the isospin symmetry breaking (ISB) terms in $\mathcal{H}$ because they contribute to $E_{\rm IAS}$ [cf. Eq. (5)]. In other words, the IAS and DIAS energies are a special filter for the terms in the Hamiltonian that break isospin symmetry (Coulomb and the small contributions from the strong force), while the isospin-conserving part of $\mathcal{H}$ does not contribute.

In order to evaluate the quartic and double commutators, we take the Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + V + V_C + V_{\rm ISB}\,, \quad (7)$$

where V is the spin- and isospin-dependent interaction, V_C is the Coulomb interaction, and $V_{\rm ISB}$ is an ISB effective interaction originating from the nuclear strong force. From Eq. (6), we can derive the relation between the DGTR and DIAS energies as,

$$E_{\rm DGTR} - E_{\rm DIAS} = \frac{\langle 0|[G_+^2,[V,G_-^2]]|0\rangle}{2(N-Z)(N-Z-1)}, \quad (8)$$

since $[G_+^2,[V_C+V_{\rm ISB},G_-^2]] = [T_+^2,[V_C+V_{\rm ISB},T_-^2]]$. In order to evaluate the energy difference between $E_{\rm DGTR}$ and $E_{\rm DIAS}$, we adopt the separable interaction[1)]

$$V = \sum_i^A \kappa_{ls}\mathbf{l}(i)\cdot\mathbf{s}(i) + \frac{1}{2}\frac{\kappa_\tau}{A}\sum_{i\neq j}^A \tau(i)\cdot\tau(j) + \frac{1}{2}\frac{\kappa_{\sigma\tau}}{A}\sum_{i\neq j}^A(\sigma(i)\cdot\sigma(j))(\tau(i)\cdot\tau(j)), \quad (9)$$

where κ_{ls} is the one-body spin-orbit coupling strength, while κ_τ and $\kappa_{\sigma\tau}$ are the coupling strengths of the residual two-body interactions in the isospin and spin-isospin channels, respectively. The average energy of the GTR minus that of the IAS is expressed as

$$E_{\rm GT} - E_{\rm IAS} = \frac{\langle 0|[G_+,[V,G_-]]|0\rangle}{(N-Z)} = -\frac{4}{3}\frac{\kappa_{ls}}{N-Z}\left\langle 0\left|\sum_i^A \mathbf{l}(i)\cdot\mathbf{s}(i)\right|0\right\rangle + 2(\kappa_{\sigma\tau}-\kappa_\tau)\frac{N-Z}{A}.$$

The energy difference between DGTR and DIAS (8) is expressed in a similar manner as

$$E_{\rm DGTR} - E_{\rm DIAS} - \left(1+\frac{N-Z}{N-Z-1}\right)(E_{\rm GT}-E_{\rm IAS}) = \frac{4}{3}\frac{\kappa_{ls}\left\langle 0|\sum_i^A \mathbf{l}(i)\cdot\mathbf{s}(i)|0\right\rangle}{(N-Z)(N-Z-1)} - 6(\kappa_{\sigma\tau}-\kappa_\tau)\frac{1}{A}\frac{N-Z}{N-Z-1}\,.$$

Quantitative discussions of the above formulas are reported elsewhere.

Reference

1) A. Bohr, B. R. Mottelson, *Nuclear Structure Vol.I* (World Scientific, 1975).

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