Nuclear charge radii with a trained feed-forward neural network†

D. Wu,[*1] C. L. Bai,[*1] H. Sagawa,[*2,*3] and H. Q. Zhang[*4]

During the last few decades, theoretical and experimental studies of the isotopic changes of ground- and low-lying states have been performed intensively to elucidate the evolution of the shell structure, shape coexistence phenomena, and shape transitions. In particular, charge radii and electromagnetic moments are very sensitive quantities from which precise information of the nuclear structure can be extracted.

Machine learning (ML) is one of the most popular algorithms for dealing with complex systems owing to its powerful and convenient inference abilities. The neural network, which is an algorithm of machine learning, has been widely used in different fields: artificial intelligence (AI), medical treatment, and physics of complex systems. In this work, we attempt to train a ML model for a description of the nuclear charge radii based directly on some experimental or quasi-experimental data, such as proton and neutron numbers, shell effect, and deformation. We search for any other physical quantities that are correlated to the charge radii.

To this end, we employ a standard fully connected feed-forward neural network (FNN), which can build a complex mapping between the input space and output space through multiple compounding of simple nonlinear functions. The FNN is a multilayer neural network with an input layer, hidden layers, and an output layer. The structure of the neural network is labelled as $[N_1, N_2, \ldots, N_n]$, where N_i denotes neuron numbers of the ith layer and $i = 1$ and n represent the input and output layer, respectively. In this study, we adopt the input layer $N_1 = 3$ and the output layer $N_n = 1$. The model is trained with the input data set of proton number Z, neutron number N, the excitation energy of the first 2^+ state $E_{2_1^+}$, and the symmetry energy. The deformation and shell effects on charge radius are included by the $E_{2_1^+}$ values. We adopt as the data set all the nuclei for which the experimental values of both $E_{2_1^+}$, and charge radii are available. The data set includes 347 nuclei in total. As the data set is not large enough, we must choose a small network structure, which includes 44 neurons and involves 201 parameters.

In the present ML study, all the charge radii of Ca, Sm, and Pb isotopes are included in the testing set to check the prediction power of models. The model reproduces well not only the slope of isotopic dependence, but also the kink of charge radii at the magic numbers $N = 82$ of Sm isotopes and $N = 126$ of Pb isotopes. The obtained charge radii of Ca isotopes with and without symmetry energy input are shown in Fig. 1. Experimental results taken from Refs. 1), 2) show a sharp kink structure at $N = 28$ and a peak between two closed shells at $N = 20$ and 28, followed by a rapid increase after $N = 28$. This figure indicates that the symmetry energy input is critical for the qualitative and quantitative description of the Ca isotopes.

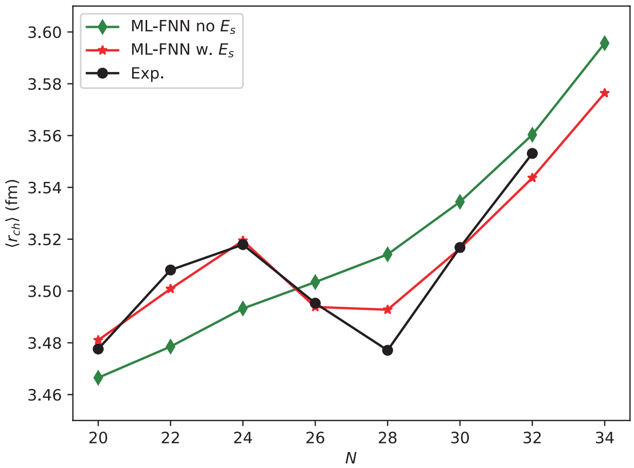

Fig. 1. Charge radii of Ca isotopes calculated by the model trained with and without taking the symmetry energy input into account. The ML results obtained with and without the symmetry energy input are labelled by the red stars and green diamonds, respectively. The experimental data taken from Refs. 1), 2) are labelled by the filled black circles.

We also perform Hartree-Fock-Bogolyubov (HFB) calculations for the radii of Ca isotopes. The HFB calculations show a clear correlation between the symmetry energy and charge radii in Ca isotopes as far as the absolute magnitude is concerned. Whereas the ML shows that the symmetry energy input has a remarkable effect on the precise descriptions of charge radii of Ca isotopes including the kink structure, the HFB calculation shows that the symmetry energy changes the absolute magnitude of charge radii, but the kink structure at $N = 28$ is not well reproduced. The physical implication of the present successful ML study remains an open question and needs to be studied in the future.

References
1) R. F. Garcia Ruiz *et al.*, Nat. Phys. **12**, 594 (2016).
2) I. Angeli, K. P. Marinova, At. Data Nucl. Data Tables **99**, 69 (2013).

† Condensed from the article in Phys. Rev. C. **102**, 054323 (2020)
*1 College of Physics, Sichuan University
*2 RIKEN Nishina Center
*3 Center for Mathematics and Physics, University of Aizu
*4 China Institute of Atomic Energy