Verification of the QED tenth-order electron $g-2$: Diagrams without a fermion loop

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A comparison between the measured value of the electron anomalous magnetic moment and its theoretical prediction is the most stringent test of the quantum electrodynamics (QED) and, hence, the standard model (SM) of elementary particles. The measurement was performed by trapping a single electron in a cylindrical Penning trap. The deviation of the g value of the electron from Dirac's prediction, $a_e = (g-2)/2$, was determined as[1)]

$$a_e[\text{expt.}] = 0.001\ 159\ 652\ 180\ 73\ (28)\ [0.24\ \text{ppb}]. \quad (1)$$

A new experiment aiming at a 20-fold improvement in the uncertainty is in progress.

To match the precision of measurement, theory must take into account small but non-negligible contributions from hadronic and electroweak interactions. The QED contribution is dominant and required up to the tenth order of perturbation theory because $(\alpha/\pi)^5 \sim 6.8\times10^{-14}$, where α is the fine-structure constant. By now, all terms up to the eighth order have been firmly established. The tenth-order term was obtained only by one group and has not yet been cross-checked.[2)] To theoretically predict a_e, a value of α is needed. Recently, two very precise values of α became available, both of which were determined using atom interferometry. The Cs[3)] and Rb[4)] atom experiments yielded

$$\alpha^{-1}(\text{Cs}) = 137.035\ 999\ 046\ (27) \quad [0.20\ \text{ppb}], \quad (2)$$
$$\alpha^{-1}(\text{Rb}) = 137.035\ 999\ 206\ (11) \quad [81\ \text{ppt}], \quad (3)$$

respectively. The difference is about $5.5\,\sigma$. A new Cs experiment has already started to reveal one more digit of α. We then have two SM predictions of a_e:

$$a_e[\text{theory}: \alpha(\text{Cs})] = 0.001\ 159\ 652\ 181\ 62\ (23), \quad (4)$$
$$a_e[\text{theory}: \alpha(\text{Rb})] = 0.001\ 159\ 652\ 180\ 265\ (94), \quad (5)$$

where the uncertainty is solely governed by that from α in both cases. Note that the discrepancy between measurement and theory is positive in sign for α(Cs), while it is negative for α(Rb).

Considering the accuracy required for future experiments, the tenth-order QED term must be carefully reexamined. A total of 12,672 Feynman vertex diagrams contributes to the tenth-order term. Among them, 6,354 diagrams without a fermion loop form a gauge-invariant set called Set V. This is the most difficult set to evaluate, and the uncertainty of the QED contribution to a_e entirely originates from Set V.

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So far, two independent numerical evaluations of Set V have been executed by two groups.[5,6)] There is, however, 4.8 σ tension between the results. The difference is still negligible for the current precision of a_e but will certainly become crucial in the near future.

In Ref. 5), numerical integration was performed on 389 self-energy-like diagrams each, which are concatenations of nine vertex diagrams via the Ward-Takahashi identity. In contrast, 3,213 integrals derived from vertex diagrams were evaluated in Ref. 6). Because the renormalization schemes used to construct integrands are different, the numerical result of one integral of the former does not coincide with the sum of the corresponding nine integrals of the latter. The direct comparison of numerical results of integrals is nontrivial.

The gap can be filled by calculating the difference in vertex renormalization constants, δL, of two renormalization schemes for every vertex diagram. The difference in numerical results of Set V integrals can be expressed in terms of these δL's and lower-order contributions of the anomalous magnetic moment.

We numerically computed all the necessary δL's that are derived from 1 second-, 4 fourth-, 28 sixth-, and 269 eighth-order vertex diagrams. The computation took approximately 120,000 hours in total. This is a small and quick calculation compared with the numerical evaluation of Set V integrals.

Diagram-by-diagram comparison was performed for the fourth-, sixth-, and eighth-order cases, and our verification method worked very well. For the tenth order, we examined numerical results representing the 2,232 Set V vertex diagrams that have no self-energy subdiagram and correspond to 135 integrals of self-energy-like diagrams. No inconsistency was found in any of the 135 integrals between Refs. 5) and 6). The investigation of the remaining 4,122 diagrams that have at least one self-energy subdiagram is in progress and will be completed soon.

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