Radiative corrections to Landau levels of a single electron revisited

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The deviation of the g value of the electron from Dirac's prediction, $a_e \equiv (g-2)/2$, is known as the anomalous magnetic moment. A comparison between the measured value of a_e and its theoretical prediction provides the most stringent test of quantum electrodynamics (QED) and, hence, the standard model of elementary particles.

The value of a_e is experimentally determined by trapping a single electron in a Penning trap and by measuring the difference in energy between different eigenstates. The latest measurement leads to[1)]

$$a_e = 1\ 159\ 652\ 180.73\ (28) \times 10^{-12} \quad [0.24 \text{ ppb}]. \quad (1)$$

A new experiment aiming at a 20-fold improvement of the uncertainty is in progress.[2)]

Considering the accuracy to be realized in future experiments, we revisited the theoretical background of the measurement. In a Penning trap, an electron is confined in the xy plane owing to the static constant magnetic field B along the z direction. Vertical confinement along the z direction is realized by the hyperbolic potential of the quadrupole electric field E. The latter effect is so small that we can ignore the E field when radiative corrections of QED to the energy levels of an electron in a Penning trap are considered.

The energy eigenvalues of an electron in the static constant magnetic field B are well known as the Landau levels in the study of quantum mechanics when its velocity v is much less than the speed of light c (see Fig. 1). The energy eigenvalue of the state with a principal quantum number $n \geq 0$ of a simple harmonic oscillator and a spin direction $\sigma_z = \pm 1$ is given by

$$E(n, \sigma_z) = \hbar\omega_c \left(n + \frac{1}{2} - \frac{g}{4}\sigma_z \right), \quad (2)$$

where $\omega_c = eB/(mc)$ with $eB > 0$ is the cyclotron frequency of an electron. The spin-flipping and anomalous frequencies are defined as $\omega_s = (g/2)\omega_c$ and $\omega_a = \omega_s - \omega_c$, respectively. The anomalous magnetic moment a_e is obtained as the ratio $a_e = \omega_a/\omega_c$.

In the early literature,[3,4)] the radiative correction to the Landau levels due to one virtual photon spanning over an electron was calculated using the electron propagator in an external magnetic field. In the weak magnetic field approximation, the correction was obtained as

$$\Delta E(n, \sigma_z) = \frac{\alpha}{2\pi} m \left[-\sigma_z \frac{eB}{2m^2} + \left(\frac{eB}{m^2}\right)^2 \left(\frac{4}{3} \ln \frac{m^2}{2eB} - \frac{13}{18} \right) + \mathcal{O}\left(\left(\frac{eB}{m^2} \right)^3 \right) \right], \quad (3)$$

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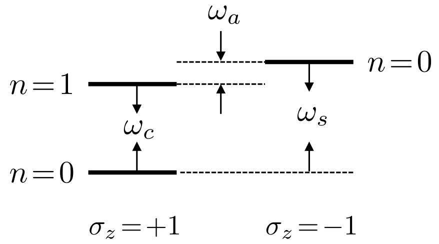

Fig. 1. Energy levels of an electron in a static constant magnetic field, known as the Landau levels. The frequency ω_c is the cyclotron frequency due to the Lorentz force acting on a charged particle. The frequency ω_s is the spin-flipping frequency of an electron with spin 1/2. The eigenvalue σ_z is the direction of the spin. Two frequencies $\omega_a \equiv \omega_s - \omega_c$ and ω_c are measured to determine the anomalous magnetic moment a_e.

with $c = 1$ and $\hbar = 1$. The first term proportional to σ_z is due to the Schwinger term of a_e. In realistic experiments, the magnetic field B is about 5 T, and we find $eB/m^2 \sim 1.1 \times 10^{-9}$, which is bigger than the current precision of a_e, 0.24×10^{-9}. The second term, however, does depend on neither n nor σ_z; thus, it does not contribute to the transition frequencies between different eigenstates, which are to be measured. The third term depends on both n and σ_z, but its magnitude is negligible. Thus, for the current precision of a_e, we need not consider the QED binding corrections to the Landau levels at all.

For the ongoing measurement of a_e, the fourth-order QED binding corrections to the Landau levels might become relevant. In order to investigate the α^2 correction, we have started reinterpreting (3) in terms of NRQED,[5)] which is suitable to describe QED corrections to non-relativistic systems. As expected, the second term of (3) is an analogue of the Lamb shift of a Coulomb-bound system. Applying Bethe's method of NRQED, we should be able to reproduce not only the logarithmic enhancement term but also the finite term of (3). Once the mechanism of QED corrections to the Landau levels is clarified, we expect that the α^2 correction will not be difficult to determine completely.

References

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