Does the second-order operator in the adiabatic expansion contribute to the collective mass?

K. Sato[*1,*2]

The adiabatic self-consistent collective coordinate (ASCC) method[1)] is a practical method for describing large-amplitude collective motion in atomic nuclei with superfluidity and an advanced version of the adiabatic time-dependent Hartree-Fock-Bogoliubov (HFB) theory. According to the generalized Thouless theorem, the state vector in the ASCC theory can be written in the form

$$|\phi(q,p)\rangle = e^{i\hat{G}(q,p)}|\phi(q)\rangle,$$

where $\hat{G}(q,p)$ is a linear combination of the $a^\dagger a^\dagger$ and aa terms. We shall call the $a^\dagger a^\dagger$ and aa terms as A-terms, and the $a^\dagger a$ and $aa^\dagger$ terms as B-terms. Recently, the ASCC theory including a second-order collective operator has been proposed.[2)] There, $\hat{G}(q,p)$ is expanded as

$$\hat{G}(q,p) = p\hat{Q}^{(1)}(q) + \frac{1}{2}p^2\hat{Q}^{(2)}(q). \tag{1}$$

In the conventional ASCC theory, only the first-order collective operator $\hat{Q}^{(1)}$ is included. However, as shown in Refs. 2) and 3), the second-order collective operator $\hat{Q}^{(2)}$ is involved in the moving-frame equations of motion. Moreover, $\hat{Q}^{(2)}$ contributes to the collective mass as

$$\begin{aligned} B(q) &= -\langle\phi(q)|[[\hat{H},\hat{Q}^{(1)}],\hat{Q}^{(1)}]|\phi(q)\rangle \\ &\quad +\langle\phi(q)|[\hat{H},i\hat{Q}^{(2)}]|\phi(q)\rangle. \end{aligned} \tag{2}$$

It is worth mentioning that the second term on the right-hand side gives a contribution of the same order as the first term.

The fundamental equations in the ASCC theory consist of the moving-frame HFB and quasiparticle random phase approximation (QRPA) equations and the canonical-variable conditions, which are derived from the invariance principle of the Schrödinger equation and the canonicity conditions, respectively. The former is the equation of motion, and the latter the conditions for the collective variables to be canonical. In the conventional ASCC theory, only the canonical-variable conditions of $O(1)$ and $O(p)$ have been taken into account, while the equations of motion up to $O(p^2)$ are solved to determine the state vector and collective operators, from which the collective Hamiltonian is calculated. In Ref. 3), the ASCC theory including the second-order collective operator $\hat{Q}^{(2)}$ was successfully applied to the Lipkin model, and it was shown that the inclusion of $\hat{Q}^{(2)}$ improves the agreement with the exact solution.

*1 Faculty of Education, Kochi University
*2 RIKEN Nishina Center

We consider the case where the pairing correlation is not taken into account, and the Hamiltonian does not include the three-body interaction. Then, we shall show that the second-order collective operator $\hat{Q}^{(2)}$ does not directly contribute to the collective mass if the second-order canonical-variable condition

$$\langle\phi(q)|[\hat{Q}^{(1)},\hat{Q}^{(2)}]|\phi(q)\rangle = 0 \tag{3}$$

is imposed.[2,3)] Noting that $\hat{Q}^{(2)}$ is written in terms of A-terms only and that the Hamiltonian can be written in terms of A-terms, B-terms, and normally ordered quartic terms, one can easily see that

$$\langle\phi(q)|[\hat{H},i\hat{Q}^{(2)}]|\phi(q)\rangle = \langle\phi(q)|[\hat{H}_A,i\hat{Q}^{(2)}]|\phi(q)\rangle. \tag{4}$$

Here, $\hat{H}_A$ denotes the A-part of $\hat{H}$.

From the moving-frame Hartree-Fock equation, we obtain

$$\hat{H}_A = \partial_q V\hat{Q}^{(1)}. \tag{5}$$

By substituting Eq. (5) into Eq. (4), we obtain

$$\begin{aligned} &\langle\phi(q)|[\hat{H},i\hat{Q}^{(2)}]|\phi(q)\rangle \\ &\quad = i\partial_q V\langle\phi(q)|[\hat{Q}^{(1)},\hat{Q}^{(2)}]|\phi(q)\rangle = 0 \end{aligned} \tag{6}$$

if the second-order canonical-variable condition in Eq. (3) is met. Then, the inertial mass $B(q)$ is

$$B(q) = -\langle\phi(q)|[[\hat{H},\hat{Q}^{(1)}],\hat{Q}^{(1)}]|\phi(q)\rangle. \tag{7}$$

The second-order collective operator $\hat{Q}^{(2)}$ does not contribute to the inertial mass directly, but it can contribute only through the equations of motion. $\hat{Q}^{(2)}$ contributes to the first- and second-order moving-frame equations of motion, which may affect the state vector $|\phi(q)\rangle$.

Here, we have concentrated on the case without the pairing correlation. When the pairing correlation is taken into account, the $\hat{Q}^{(2)}$ term in the inertial mass in Eq. (2) does not vanish even if the condition in Eq. (3) is imposed. Before ending this report, we add one remark. It is not trivial whether one should/can include the second-order canonical-variable condition in Eq. (3) in the set of fundamental equations of the ASCC theory. If it is imposed in addition to the first-order canonical-variable conditions, there may be too many conditions to determine the unknown quantities, and the problem may be overdetermined. This point will be investigated and reported in a future publication.

References

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